A Model of Magnetic Vector Potential Based on the Principles of Quantum Mechanics

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Abstract

The aim of the paper is to show that the magnetic vector potential affects both the magnitude and direction of the frequency of the Schrödinger wave function of the quantum entity moving in the region where the potential is not zero while the magnetic field may be equal to zero. The change in the frequency results in a change in the quantum entity wave function phase, the latter being observed in the experiments where the effects of magnetic vector potential on quantum entities are studied. The model of magnetic vector potential discussed in this paper is based on the properties of the virtual particles created by quantum entities in the physical vacuum. Besides, the quantum mechanical concept of the existence in the physical vacuum of quantum harmonic oscillators with zero-point energy is taken into account. It is shown that the oscillation frequency of quantum harmonic oscillators determines the magnitude of magnetic vector potential. An equation has been derived that relates the change in the frequency of the Schrödinger wave function (and hence the change in the energy) of quantum entity to the value of magnetic vector potential affecting the quantum entity.

Keywords: magnetic vector potential; virtual particles; quantum harmonic oscillators; zero point energy; wave function frequency; wave function phase; quantum mechanics.

Introduction

In classical electrodynamics the magnetic field of induction $\mathbf{B}$ is determined [1] by equation:

$$\mathbf{B} = \text{curl}\mathbf{A} \tag{1}$$

where $\mathbf{A}$ is a magnetic vector potential. The presence of a non-zero magnetic vector potential in a region of space is not a sufficient condition of existence of magnetic field $\mathbf{B}$ in the region. In the screening of magnetic field the following may take place: $\mathbf{A} \neq 0$, but $\mathbf{B}=0$. This case is referred to as the field-free vector potential [1]. Magnetic vector potential has a physical meaning of its own. In 1949 Erenberg and Siday predicted the ability of magnetic vector potential to influence directly the characteristics of quantum entities even though there is no electromagnetic field at the location of the entities [2]. In 1959 the possibility of such an effect was considered by Aharonov and Bohm [3]. Subsequently, a great number of experiments have been conducted which confirmed the theoretical predictions [4]. In general, these experiments were as follows (see Fig. 1): the beam of quantum entities emitted by a source is split into two beams: $C_1$ and $C_2$. Beam $C_1$ propagates through the region where $\mathbf{A} = 0$. Beam $C_2$ passes through an energized toroidal solenoid. The solenoid is shielded in such a way that outside the solenoid there is no magnetic field, $\mathbf{B} = 0$, but the vector potential is present: $\mathbf{A} \neq 0$. Both beams of quantum entities arrive at the entrances of an interferometer. The interference rings obtained suggest that there is a change in the wave function phase of quantum entities passing through the region where $\mathbf{B} = 0$ and $\mathbf{A} \neq 0$.

![Figure 1](image)

Figure 1. Schematic diagram of the experiment on the study of the effects of magnetic vector potential on quantum entities. The source of quantum entities emits two beams. Beam $C_1$ propagates through the region where $\mathbf{A} = 0$. Beam $C_2$ passes through the shielded energized toroidal solenoid in the region of the field-free magnetic vector potential. Interference rings are produced by the interferometer.

In some experiments not a toroidal solenoid is used, but an indefinitely long (theoretically) solenoid. In this case the beams of quantum entities $C_1$ and $C_2$ propagate on the different sides of the solenoid, that is through the regions of field-free vector potentials of opposite sign.

In quantum mechanics, the characteristics of a quantum entity with nonzero rest mass are determined by the Schrödinger equation. The solution to this equation is the wave function, $\psi(r, t)$ ($r$ is the coordinate, $t$ is time) which is characterized by a phase and the frequency $\omega_{sh}$ defined as [5]

$$\omega_{sh} = E/h, \tag{2}$$

where $h$ is the Planck constant, $E$ is the quantum entity energy.
After de Broglie put forward a hypothesis that with every particle of nonzero rest mass a wave is associated, many efforts were focused on searching a “physical” wave that could accompany the particle: the so-called pilot wave [5, 6]. The failure of such attempts resulted in that the Born proposal has been accepted, whose main idea consists in that only the square of the absolute value of the wave function
\[ |\psi(r, t)|^2 \]
has the physical meaning: it determines the probability density of finding the particle at a point of space.

The experiments examining the effects of magnetic vector potential on quantum entities suggest that the magnetic vector potential affects the wave function characteristic that has a physical meaning. The wave function phase was taken as such a characteristic because the difference in the phases of wave functions of the interacting quantum entities was observed by interferometry. The frequency in the experiments was considered to be constant and, consequently, according to Eq. (2) the energy \( E \) was considered to be constant too. This meant that the steady-state Schrödinger equation [5] could be used for the description of the quantum entity behavior in the field of action of magnetic vector potential.

The steady-state Schrödinger equation used for the description of characteristics of a quantum entity which has mass \( m \) and is located in the region of some potential, \( V(r) \), is as follows:
\[
\Delta\psi(r, t) + \frac{2m}{\hbar^2}[E - V(r)]\psi(r, t) = 0.
\] (3)

In quantum mechanics, the solution to Eq. (3), \( \psi(r, t) \), is written as
\[
\psi(r, t) = \psi(r) \cdot f(t),
\] (4)
where \( \psi(r) \) is the coordinate part determining the wave function phase; \( f(t) \) is the time part determining the wave function frequency of the quantum entity:
\[
f(t) = \exp(-iEt/\hbar).
\] (5)

It follows from Eqs. (4) and (5) that \( |\psi(r, t)|^2 = |\psi(r)|^2 \). If to follow Born’s proposal, this equality means that the wave function phase has a physical meaning; while taking into account Eq. (4) the wave function frequency may have no physical meaning.

However, there are works [7-9] where it is shown that the wave function frequency does have a physical meaning for quantum entities with nonzero rest mass: it is the spin precession frequency of virtual particles pairs created by quantum entities in the physical vacuum. The endowment of the wave function frequency with the physical meaning enables one to describe the phenomena for which there has been no physical model so far, e.g. quantum correlations that belong to the class of phenomena called “quantum nonlocality”. The effect of magnetic vector potential on the quantum entity wave function refers to the same class of phenomena.

It will be shown in this paper that the results of the experiments on studying the magnetic vector potential can be explained by the effect of the latter on the spin precession frequency of virtual particles pairs created by quantum entities in the physical vacuum, that is, according to [7-9] on the frequency of the quantum entity wave function. The change of the wave function frequency results in a change of the wave function phase, which is observed in experiments.

It should be noted that a change in the frequency of the Schrödinger wave function of a quantum entity means as well, according to Eq. 2, a change in the energy \( E \) of the entity. Because of that, the use of the steady-state Schrödinger equation for the description of the state of the quantum entity subjected to a magnetic vector potential is improper.

The way to explain the action of magnetic vector potential is based on taking into consideration the properties of virtual particles created by the quantum entity in the physical vacuum [10]. Besides, the concept of existence in the physical vacuum of quantum harmonic oscillators with zero-point energy, accepted in quantum mechanics, is also taken into consideration [11, 12]. It is shown that the oscillation frequency of quantum harmonic oscillators determines the value of magnetic vector potential, the latter being able to affect both the magnitude and direction of the quantum entity wave function frequency.

### A. The Properties of Virtual Particles

Every moving quantum entity produces a pair of oppositely charged virtual particles, which have a mass and spin, in the physical vacuum. The virtual particle spin has the same properties as the real particle spin, hence it follows that:

a) the spin of a pair of virtual particles has no definite direction, and by the magnitude of spin the magnitude of its projection onto a preferential direction is meant; this can be interpreted as a precession of the spin about the preferential direction; the precession is characterized by the precession frequency, precession phase (angle) and angle of deflection;

b) spin correlations may take place between the spins of pairs of virtual particles.

The virtual particles pair may convert into a pair of real particles with the total spin equal to \( \hbar \). Taking into account the angular momentum conservation principle, one may suppose that the total spin \( S_v \) of a virtual particles pair, at least while converting into real particles, must be equal to \( \hbar \). But the virtual particles pair is created by a quantum entity whose spin is equal to \( \hbar/2 \), and while creating the virtual particles pair the quantum entity conserves this spin. Hence this may suggest that the physical vacuum has an intrinsic
degree of freedom, i.e. spin, which manifests itself at the creation of virtual particles. Thus $S_v$ is the total spin determining the intrinsic degree of freedom of the physical vacuum in the region where the virtual particles pair is created.

Since the virtual particle spin has the same properties as the real particle spin, spin correlations may take place between the spins. Experiments conducted with superfluid $^3$He-B showed that the spin correlations may be effected by spin supercurrent [13-15]. The spin supercurrent may emerge between the spins of the virtual particles pairs and the spins that determine the intrinsic degree of freedom of the physical vacuum. The spin supercurrent makes equal the respective characteristics of precessing spins (the deflection and precession angles), between which the spin supercurrent emerges, and thus the “propagation” of the spin precession of the virtual particles pairs takes place in the physical vacuum. It follows from the properties of virtual particles pair that not only the precession frequency, the deflection and precession angles are associated with the precessing spin, but also a mass. In Fig. 2 are shown the characteristics associated with the “propagation” of spin precession: $S_\Omega$, $\Omega$, $\theta$, $\alpha$, $m_\Omega$, $Z_\Omega$ (respectively the spin, precession frequency, deflection angle, precession phase (angle), mass, mass angular momentum). Energy $W_m$ of circular motion of mass $m_\Omega$ is determined [16] by the relation: $W_m = Z_\Omega \cdot \Omega / 2$. If $Z_\Omega = \hbar$, then:

$$W_m = \hbar \cdot \Omega / 2.$$  \hspace{1cm} (6)

![Figure 2. The characteristics of a quantum harmonic oscillator: $S_\Omega$ is spin; $\Omega$ is the precession frequency; $Z_\Omega$ is the angular momentum associated with mass $m_\Omega$; $\theta$ is the deflection angle; $\alpha$ is the precession phase (angle); ref. line is the reference line.](image)

The properties of the objects created in the physical vacuum due to “propagation” of spin precession (the existence of an oscillatory process; the proportionality of the energy of the objects to the oscillation frequency, see Eq. (6); the possibility of interaction of neighboring objects with each other) are the same as those determining the quantum harmonic oscillators with zero-point energy.

The concept of zero-point energy was developed by Albert Einstein and Otto Stern in 1913 [11]. According to this concept, it is the energy of the vacuum, which in quantum field theory is defined not as an empty space but as the ground state of the field whose features are as follows [12]: 1) the field consists of oscillators; 2) the energy of such oscillator is proportional to the oscillation frequency, the proportionality factor being $\hbar / 2$; 3) neighboring oscillators interact with each other.

Because of the above similarity of the properties, the objects created in the physical vacuum due to the “propagation” of precession of virtual particles pair spin can be referred to as “quantum harmonic oscillator”, in this paper the abbreviation QHO is used.

B. The Relationship Between the Properties of Magnetic Vector Potential and the QHO Properties

Electric current, $I$, is formed by the motion of charged quantum entities, each of the latter producing a virtual particles pair in the physical vacuum. It is shown in [17] that the direction of precession frequency, $\omega_q$, of spin of virtual particles pair created by the quantum entity is determined by the sign of electric charge of this entity: $\omega_q \uparrow \uparrow I$. The total precession frequency, $\omega_f$, of spins of virtual particles pairs created by the charges that form the current is determined as:

$$\omega_f = \omega_q I / q,$$  \hspace{1cm} (7)

where the value $I/q$ is equal to the number of the quantum entities having electric charge $q$, which form current $I$.

Because of the “propagation” of the spin precession with frequency $\omega_f$, at point $P$ which is located at a distance $r$ from the wire carrying current $I$ there would arise a QHO with the precession frequency $\Omega$:

$$\Omega \uparrow \uparrow \omega_f.$$  \hspace{1cm} (8)

If to introduce $f_\Omega (r)$ determining the dependence of $\Omega$ on $r$, $\Omega$ may be expressed as $\Omega = \omega_f f_\Omega (r)$. Using Eq. (7) and taking into account (8), the latter expression for $\Omega$ can be written as:

$$\Omega = \omega_q f_\Omega (r) I / q.$$  \hspace{1cm} (9)

Let us consider the propagation of a quantum entity through point $P$ of the physical vacuum. The quantum entity creates a virtual particles pair with the precession frequency $\omega_e$ of spin $S_e$. (We neglect the size of the virtual particles pair created by the moving quantum entity in the physical vacuum.)

The spin precession frequency $\Omega$ of QHO at point $P$ causes a change in the spin precession frequency of the virtual particles pair created by the quantum entity; the nature of the change depends on the mutual orientation of
It is shown in [7-9] that the quantum entity wave function phase is essentially the phase (angle) of precession of spin of virtual particles pair created by the quantum entity in the physical vacuum, that is
\[
\delta \phi_{\psi} = \delta \alpha ,
\]
where \( \delta \phi_{\psi} \) is a change in the wave function phase. Taking into account Eq. (10), the quantity \( \delta \phi_{\psi} \) as specified by Eq. (11) may determine the result of the above mentioned experiment (see Introduction) on interference of two beams of quantum entities.

Therefore, the creation of QHO with precessing spin in the physical vacuum may be assumed to be the physical process that is responsible for the change in the wave function phase of quantum entities passing through the region where \( B = 0 \) and \( A \neq 0 \).

Let us determine the relationship between the precession frequency \( \Omega \) and the magnetic vector potential created by electric current \( I \). The magnetic vector potential \( A \) created by element \( \varepsilon \) of a wire carrying current \( I \) is determined at the distance \( r \) from the wire (at point \( P \), see Fig. 3), provided that \( \varepsilon \ll r \), as [18]:
\[
A = \frac{I \varepsilon}{4\pi \varepsilon_0 c}.
\]

By solving simultaneous equations (9) and (12), it is possible to obtain the relationship between the value of magnetic vector potential \( A \) in a definite region of the physical vacuum and the value of precession frequency \( \Omega \) of QHO, created in that region:
\[
A = \frac{\Omega}{4\pi \varepsilon_0 c \cdot \varepsilon \cdot r \cdot f_\Omega (r)}.
\]

Solving simultaneous equations (10), (11) and (13) and expressing \( \omega_q \) in terms of the energy \( E_q \) of the charges that form the current \( I \) [7-9]:
\[
\omega_q = E_q / h
\]

it is possible to relate \( \delta \phi_{\psi} \) to \( A \) in the following way:
\[
\delta \phi_{\psi} = A \frac{4\pi \varepsilon_0 c \cdot E_q \cdot f_\Omega (r)}{q \cdot h \cdot \varepsilon} t .
\]

C. The Inversion of Spin of Virtual Particles Pair.

The magnetic vector potential can be used to invert the spin of virtual particles pair. Figure 4 illustrates how the inversion of spin, \( S_e \), of a virtual particles pair located at point \( P \) can be performed. The spin is precessing with frequency \( \omega_e \).
According to Eq. (9), the alternating electric current $I$ creates at point P a QHO with the alternating precession frequency $\Omega$ of the QHO spin. At $\Omega = \omega_e$, the inversion of spin $S_e$ may take place [16] (in Figure 4 the direction of spin after the inversion is shown by vector $S_e'$), the inversion being accompanied by consumption of energy from the source of current $I$.

![Figure 4](image)

**Figure 4.** The diagram illustrating the reversal of virtual particles pair spin (spin resonance). $\sim \Omega$ is the precession frequency of the spin of QHO created by the alternating electric current $\sim I$ at point P; $\omega_e$ is the frequency of precession of virtual particles pair spin $S_e$, $\omega_e \perp \Omega$; $S_e'$ is the inverse direction of spin $S_e$.

The above discussed way of performing the spin resonance can be used for measuring the energy of quantum entities. To this end it is necessary to scale, according to Eq. (9), the current $I$ in terms of frequency $\Omega$ and measure the value of $\Omega$ and consequently the value of frequency $\omega_e$ at the time when energy is consumed from the source of current $I$, that is in the course of spin resonance. Knowing $\omega_e$ and using Eqs. (2) and (16), it is possible to determine the energy of the quantum entity that created the virtual particles pair with spin precession frequency equal to $\omega_e$.

Note. In physics, a phenomenon is known in which the spin inversion occurs as a result of spin-spin interactions: this is the pseudomagnetic resonance. There was experimentally obtained the inversion of spin of neutrons passing through the substance with spin-polarized nuclei (that is the neutrons are travelling in the so-called nuclear pseudomagnetic field), when the neutrons are exposed to high-frequency pseudomagnetic field, whose direction is normal to the direction of the nuclear pseudomagnetic field [19].

**D. Discussion**

I. The observation of the topological Aharonov-Casher phase shift by neutron interferometry.

Figure 5 is a schematic diagram of the experiment that demonstrates the Aharonov-Casher topological phase shift [20]. Spin-polarized neutrons emitted by a source are divided into two beams. Neutrons of different beams pass on different sides of the line charge and arrive at the interferometer entrances. The electric field strength in the region where the neutrons propagate is $E_n$. Interference fringes were observed in the experiment, which suggests that there is a difference in the wave function phases of the neutrons that passed on different sides of the line charge.

![Figure 5](image)

**Figure 5.** Diagram of the experiment that demonstrates the Aharonov-Casher topological effect. $E_n$ is the electric field strength produced by the line charge in the region where the neutrons propagate; $w$ is the neutron velocity.

The difference in the phases was such as if a magnetic field acted upon a neutron in the reference frame of the neutron, the magnetic induction $B_n$ being equal to $(1/c)E_n \times w$. In the model developed in this paper, for the existence of a magnetic field and accordingly the magnetic vector potential in the frame of neutron it is necessary that the physical vacuum in the region where neutrons propagate contained QHO in the same frame. However, the line charge, which is at rest (relative to the physical vacuum), does not create QHO in the physical vacuum. Therefore, in the model there is no magnetic vector potential in the frame of reference of neutron.

To explain the topological Aharonov-Casher phase shift, let us consider the electric dipole moment $d_e$ of virtual particles pair created by a neutron in the physical vacuum [7, 17]. In the electric field of the line charge, the moment $M_n$ defined as $M_n = d_e \times E_n$ will affect the characteristics of the precession of spin of virtual particles pair created by the neutron in the physical vacuum. These characteristics, according to [7-9], are essentially the characteristics of the neutron wave function. In more detail the effect of electric field on a quantum entity due to the existence of the electric dipole moment of the virtual particles pair created by a quantum entity in the physical vacuum is discussed in [7, 17].

II. Equation (15), relating the change in the wave function phase to the magnitude of magnetic vector potential was derived under the assumption of point size of the virtual particles pair created by the quantum entity in the physical vacuum. However, the virtual particles pair is created by the quantum entity in the region whose size is equal to the de Broglie wavelength, $\lambda$, of that entity, which is determined as $\lambda = h / p$, where $p$ is the quantum entity momentum.
III. According to the findings of [7], the endowment of the physical vacuum with the properties of superfluid \(^{3}\)He-B proves to be useful for describing a lot of physical phenomena. In the model of physical vacuum with such properties, a virtual particles pair emerges in the vortex created by a moving quantum entity. Due to the Barnett effect, the precession of spin of the entities that constitute the physical vacuum is produced in vortices. As a result of spin-spin interaction the vortices diffuse in the volume of the physical vacuum and in this way quantum harmonic oscillators emerge. Thus in the model of physical vacuum possessing the properties of superfluid \(^{3}\)He-B the physical vacuum with QHO can be classified as a "vorticity" vacuum.

Conclusion

The motion of quantum entities in the form of electric current accounts for the creation of quantum harmonic oscillators in the physical vacuum. The oscillation frequency of the oscillators determines the magnitude of magnetic vector potential produced by the electric current.

The magnetic vector potential affects the frequency of the Schrödinger wave function of the quantum entity. The equation was derived relating a change in the frequency of the Schrödinger wave function of the quantum entity to the value of the vector potential that affects the quantum entity.

The effect of magnetic vector potential on the quantum entity is an energy effect. An equation determining the change in the energy of the quantum entity being affected by magnetic vector potential was derived in the paper.

By means of magnetic vector potential it is possible to produce the inversion of virtual particles pair spin, which makes it possible to measure the wave function frequency and consequently the energy of the quantum entity that created the virtual particles pair.

References

A MODEL OF MAGNETIC VECTOR POTENTIAL BASED ON THE PRINCIPLES OF QUANTUM MECHANICS

Biography

Liudmila Borisovna Boldyreva graduated from Moscow Engineering Physics Institute. She defended her PhD thesis on processing results of physical experiments. For about 30 years she has been studying the properties of physical vacuum. The results are published in her book “What does this give to physics: attributing the properties of superfluid $^3$He-B to physical vacuum?” and in the papers, part of which are referred to in this paper. At present she works as Associate Professor at the State University of Management (Moscow).

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